Optimal Distributed Voltage Regulation for Secondary Networks With DGs

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***Abstract*—An algorithm for the optimal voltage regulation of distribution secondary networks with distributed generators (DGs) is proposed in the paper. Based on the ε decomposition of the sensitivity matrix (inverse of Jacobian) obtained from the solution of the Newton-Raphson power flow problem, a large secondary network is divided into several small subnetworks. From the ε decomposition, the range of influence of each DG on the voltage of the entire network is determined. When voltage at particular nodes exceeds normal operating limits, the nearest DGs can be located and commanded to control the voltage. The control action can be coordinated using communications in a small-size subnetwork. The voltage regulation is achieved by solving a small linear programming optimization problem with an objective function that makes every DG to optimize its generation. The algorithm is tested with a model of a real heavily-meshed secondary network. The results show that the algorithm proposed in this paper can effectively control the voltage in a distributed manner. It is also discussed in the paper how to choose the value of ε for the system decomposition.**

***Index Terms*—Distributed generation, optimal distributed voltage regulation, secondary network, sensitivity matrix, ε decomposition.**

I. Introduction

NOWADAYS, distributed generators (DGs) are an emerging alternative for energy production. DGs are changing the traditional, centralized, and large scale power generation, to a distributed and small scale generation. This poses new and different challenges to the system design and operation such as: stability problems, voltage regulation issues, diverse electricity markets, and so on [1]–[4]. In this paper, a method is proposed for voltage regulation of secondary networks based on an optimal distributed approach.

The voltage regulation of distribution systems is customarily provided by on-load tap changer (OLTC) transformers installed in the substation complemented by voltage regulators in the feeders and reactive compensation. Some voltage regulators compensate for the voltage drop on the lines to control the voltage at a certain distance. These control devices cannot, however, react fast enough in emergency conditions [5]. Furthermore, in a distribution network with DGs, the settings of these traditional devices are different from those of traditional systems without DGs [5]. Performing voltage control based on DGs was proposed in [5], [6], [7], [8], [9], [10], [11], [12], [13], and [14]. In [4] and [12] an investigation was performed on the linear relationships between DG and distribution network voltage. An intelligent method using DGs for voltage control in a distribution network was proposed in [6] and [8]. In [10] and [13] not only a voltage control method with a single DG was presented, but also offered a method for coordinating DGs and traditional voltage control devices using a centralized system to minimize losses. A method to prevent the voltage at the point of common coupling (PCC) from exceeding the upper operating limit by changing the reactive power generation of a DG was proposed in [9] and [14]. All these references mostly discuss how a single DG can regulate local voltage by adjusting its generation.

There have been a few publications proposing coordinated or distributed voltage regulation methods using multi-DGs. For example, [5] puts forward a multi-DG based optimal voltage control method, which decomposes the system voltage control problem into small subproblems and solves them using multiagent system (MAS). In [11], the surplus reactive capacity of PV type DGs is used to manage the line voltage via cooperative control. These existing methods require a communication system connecting all nodes whether they have DG installed or not. The need for global communication systems seriously limits the benefits of coordinated control to small distributed systems. Additionally, controlling all node voltages creates a heavy computational burden in large systems. All previous regulation studies were made on radial feeders; there are no publications dealing with voltage regulation for highly-meshed secondary networks.

Among contemporary power distribution networks around the world, some of the networks allow DGs to work in the power factor control (PFC) mode and some in the unity power factor (UPF) control mode. In this paper, an optimal distributed voltage regulation of distribution secondary networks with DGs is proposed that is suitable for both the PFC and the UPF control modes. The objective of the proposed method is to keep the voltage profile within the normal operation limits (0.95 to 1.05 p.u.). The method is based on optimal generation for every involved DG from the economical operation viewpoint, to be precise: a) For DGs operating in the UPF mode, the method always maximizes the active power output of every involved DG; b) for DGs operating in the PFC mode, the method always minimizes the reactive power generation adjustments of every involved DG. Both functions are performed optimally regardless whether the voltage is higher or lower than the normal operating limits.

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The proposed method eliminates the need for global communications. Communication systems are only needed within the boundaries of the small subnetworks that are created by the application of the ε decomposition to the sensitivity matrix (inverse of the Jacobian matrix of the Newton-Raphson power flow). The ε decomposition of the sensitivity matrix keeps the strong couplings between DGs and breaks the weak couplings. Therefore, in order to control network voltage, each DG only needs to communicate with DGs that are in the same subnetwork. This effectively decomposes the network voltage regulation problem into several small subnetwork voltage regulation problems.

This paper is organized as follows. In Section II, the linear relationship between DG generation and network voltage profile together with the distributed control scheme are presented. In Section III, the test network is introduced. In Section IV, the distributed voltage regulation algorithm is analyzed and a way to choose the value of ε is proposed. Conclusions are given in Section V. A demonstration of the ε decomposition algorithm is given in the Appendix.

II. Voltage Control Based on DGs

A. Relationship Between DG Output and Voltage Profile

From the Jacobian matrix of the Newton power flow, the sensitivity matrix Λ can be obtained which describes a linear relationship between variations of DG (active power and reactive power) and voltage changes as follows:

$$\begin{pmatrix} \Delta\theta \\ \Delta V \end{pmatrix} = \begin{pmatrix} \Lambda_{\theta P} & \Lambda_{\theta Q} \\ \Lambda_{VP} & \Lambda_{VQ} \end{pmatrix} \begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix} \tag{1}$$

with

$$\Lambda = \begin{pmatrix} \Lambda_{\theta P} & \Lambda_{\theta Q} \\ \Lambda_{VP} & \Lambda_{VQ} \end{pmatrix}. \tag{2}$$

With the initial voltage V_0 and the reference voltage V_r at a certain node, we can compute the DG reactive power adjustment $x_Q = Q_r - Q_0$, where Q_r and Q_0 are reactive power output after and before voltage regulation, respectively, or the active power adjustment $x_P = P_r - P_0$, where P_r and P_0 are active power output after and before voltage regulation, respectively to control voltage from V_0 to V_r using

$$V_r = V_0 + \Lambda_{VQ} \cdot x_Q + \Lambda_{VP} \cdot x_P. \tag{3}$$

B. The ε Decomposition

The ε decomposition [15], [16] is used for the partition of a large system into weakly coupled subsystems. We apply this method for the decomposition of the sensitivity matrix Λ in (2). Let us take the submatrix Λ_{VP} as an example

$$\Lambda_{VP} = \Lambda'_{VP} + \varepsilon \cdot R \tag{4}$$

where Λ'_{VP} is the sensitivity matrix with elements larger than ε, which means only strong couplings, and $\varepsilon \cdot R$ is the residue matrix which describes weak couplings. All elements of R are smaller than or equal to one. In addition to quantitatively describing the coupling between DGs and nodes, Λ'_{VP} also describes the influence range of every DG and topology of the "new" network which neglects weak couplings. For matrix Λ'_{VP}, a permutation matrix P can be obtained to convert Λ'_{VP} to a $\widetilde{\Lambda_{VP}}$ as $\widetilde{\Lambda_{VP}} = P^T \Lambda'_{VP} P$, where $\widetilde{\Lambda_{VP}} = diag\{\Lambda_{11}, \Lambda_{22}, \ldots, \Lambda_{NN}\}$ is block-diagonal [15], [16]. And in $\widetilde{\Lambda_{VP}}$, each block stands for the topology of each subnetwork. However, in this paper, it is convenient to obtain the subnetworks topology described by matrix Λ'_{VP} using the Deep First Search (DFS) algorithm [17]. An example of the decomposition is shown in the Appendix.

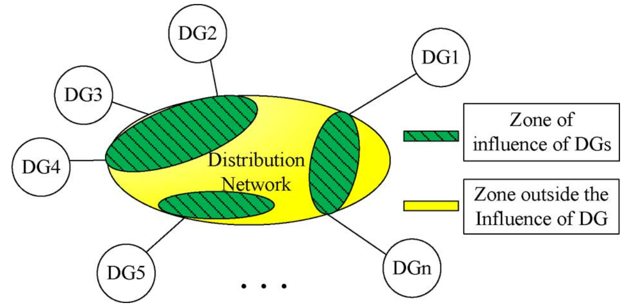

Fig. 1. Zone of influence of DGs.

The same decomposition can be performed for Λ_{VQ} to obtain the range of influence of a particular DG (or a group of DGs). The results are represented schematically in Fig. 1. It is possible to have intersections among the ranges of influence of DGs and at the same time there could be areas that cannot be covered by any DG. The range of influence can be expanded or contracted by changing the threshold value.

Because the ε decomposition breaks the system into small isolated subnetworks, it allows dividing the system communications into small subnetworks as well.

C. Optimal Distributed Voltage Control

When voltage violations happen (variations larger than $+/-5\%$), optimal adjustments to the distributed generator generation can be calculated with the information of the sensitivity matrix using a linear programming (LP) method.

When all DGs operate in the PFC mode, one can optimally control the voltage by minimally increasing or decreasing the involved local DGs' reactive power generation. Then the objective function is:

to control voltage from higher than 1.05 p.u.

$$\text{Max: } Min\{x_i\}; \tag{5}$$

to control voltage from lower than 0.95 p.u.

$$\text{Min: } Max\{x_i\}; \tag{6}$$

both subject to the following constraints:

$$\begin{cases} V_l \leq V_0 + \Lambda_{VQ} \cdot x \leq V_u \\ x \leq Q_{surplus} \end{cases}; \tag{7}$$

where x_i is the i-th adjustment of reactive power for the i-th involved DG, x is the vector of all x_i. When controlling

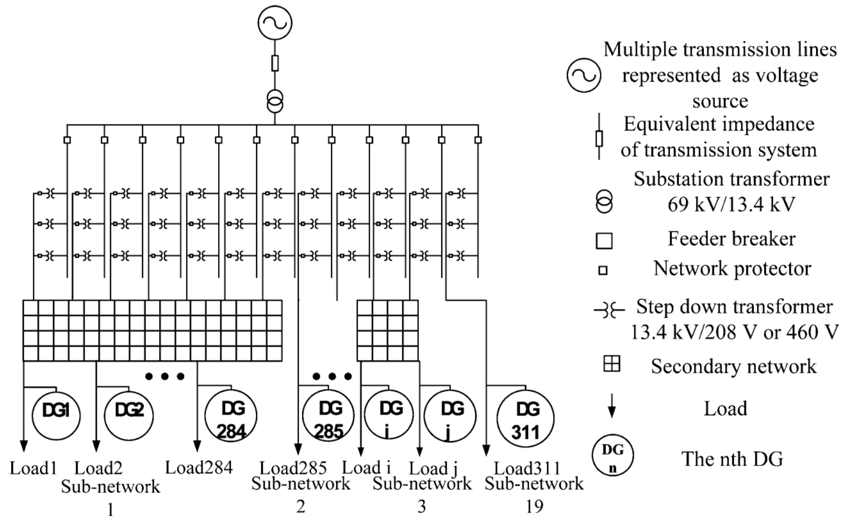

Fig. 2. Structure of the system under study.

voltage from higher than 1.05 p.u., x_i is a negative number, which means a decrease in the reactive power generation. When controlling voltage from lower than 0.95 p.u., x_i is a positive number, which means an increase in the reactive power generation.

When all DGs operate in the UPC mode, DGs can only work at unity power factor, i.e. generating active power only. Hence, one can control the voltage by decreasing local DGs' generation when the voltage is beyond the upper limit of normal operation. In the opposite case, when voltage is beyond the lower limit of normal operation, the control is achieved by increasing local DGs' generation. These cases can be modeled by LP with the following objective function:

$$\text{Max: } Min\{x_\text{i}\} \tag{8}$$

subject to

$$\begin{cases} V_l \leq V_0 + \Lambda_{VP} \cdot x \leq V_u \\ x \leq P_{surplus} \end{cases} \tag{9}$$

where x_i is the i-th adjustment of active power of i-th involved DG, and x is the vector of x_i. When controlling voltage from higher than 1.05 p.u., x_i is a negative number, which means a decrease in the active power generation. When controlling voltage from lower than 0.95 p.u., x_i is a positive number, which means an increase in the active power generation.

In the above two LP problems, V_l is the lower bound of voltage, i.e. 0.95 p.u., V_u is the upper bound of voltage, i.e. 1.05 p.u.. $P_{surplus}$ and $Q_{surplus}$ are surplus capacities of DG, i.e. the capacities available in the DG between the initial generation point and its rating.

III. Simulation Tests

In this section, first, the study network is described. Then, the simulation steps are shown, which include the ε decomposition of the network, the allocation of DGs range of influence, and the statement and solution of the LP problem. Finally, the simulation results are analyzed.

A. Simulation Test System

A simulation test of the proposed control method has been performed on a model of a real heavily-meshed distribution secondary network which has 2083 nodes (1043 nodes at 13.8 kV at primary feeders, the remaining 1040 nodes are at 480 V or 216 V composing the secondary network), 311 PQ loads, and 224 network transformers (13.8 kV to 216 V or 480 V). The total light-load is 54.88 MVA at 0.89 power factor. The secondary network contains 19 subnetworks which are isolated on the secondary side from each other. The detailed information about this distribution system is shown in Fig. 2.

It can be seen from Fig. 2 that all network transformers in the first subnetwork are connected on the secondary side, creating a heavily meshed structure (containing 284 PQ loads). The other 18 subnetworks are spot networks, i.e. only a few transformers supply one or two loads. Network protectors, installed on the secondary side of each network transformer, are used to prevent reverse active power flow from the secondary side to the primary side of the distribution network. If reverse active power flow is detected the network protector will trip to disconnect the transformer from the secondary. Assuming that DGs are located at the load nodes, there are 311 possible DGs in the system. It follows from the above discussion that there are three kinds of constraints for voltage control: (1) node voltages should be in their normal limit (+/−5%); (2) voltage angles of the primary side of network transformers should lead those of the secondary side (preventing reverse active power flow through network transformer); and (3) generation adjustment should not exceed surplus capability of each DG. The second constraint can be expressed as follows:

$$0 \leq \theta_{p0} + \Lambda_{\theta_p Q} \cdot x - (\theta_{s0} + \theta_{shift} + \Lambda_{\theta_s Q} \cdot x) \tag{10}$$

$$0 \leq \theta_{p0} + \Lambda_{\theta_p P} \cdot x - (\theta_{s0} + \theta_{shift} + \Lambda_{\theta_s P} \cdot x) \tag{11}$$

where θ_{p0} and θ_{s0} are the initial values of primary side and secondary side voltage angles of the network transformers, $\Lambda_{\theta_p Q}$ and $\Lambda_{\theta_p P}$ are the sensitivity matrices that describe the relationship between the adjustment of the reactive power and the primary side and secondary side voltage angle, $\Lambda_{\theta_s Q}$

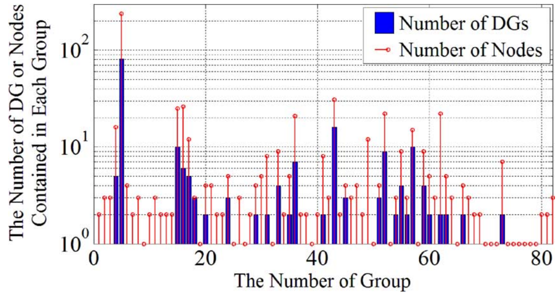

Fig. 3. Results of the application of the ε decomposition of $\Lambda_{VQ}, \varepsilon = 0.012$.

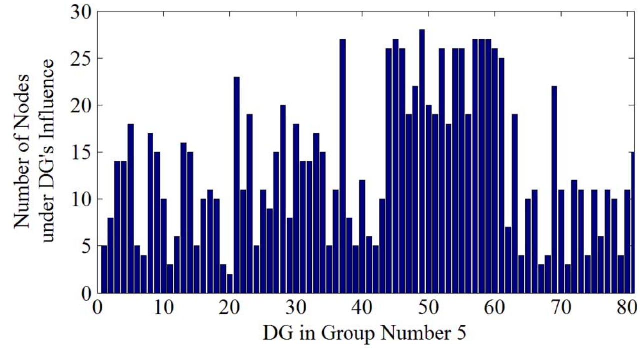

Fig. 4. Number of nodes in the influence range of each DG of group No. 5.

and $\Lambda_{\theta_s P}$ are sensitivity matrices that describe relationships between adjustment of active power and primary side and secondary side voltage angle. Equation (10) is for the DGs in PFC mode whereas (11) is for the DGs in UPF mode.

We can also determine the range of influence of the 311 DGs' to the 224 network transformers' voltage angles by using the same decoupling method for the sensitivity matrix. This is achieved with $\varepsilon = 0.007$ for the $\Lambda_{\theta P}$ and $\Lambda_{\theta Q}$ sensitivity matrices. The results show that when DGs work in UPC mode, there are 221 DGs that can affect network transformers' voltage angles. Among these 221 DGs, there are 94 DGs that can affect 2 network transformers, 56 DGs that can affect 3 network transformers, 18 DGs that can affect 4 network transformers, 3 DGs that can affect 5 network transformers, and no DG can affect more than 5 network transformers. When DGs work in PFC mode, there are 59 DGs that can affect network transformers' voltage angles. Among these 59 DGs, there are 3 DGs that can affect 2 network transformers, and no DG can affect more than 2 network transformers. For a given DG, the information on how the voltages of the network transformers are affected is stored in the DG's control module for the voltage control.

B. Results of the ε Decomposition on the Sensitivity Matrix

Based on the ε decomposition of the sensitivity matrix Λ of the network under study, information on DG's influence range and DGs grouping can be obtained. The sensitivity matrix is obtained from the load-flow program. It is a 4164×4164 matrix. In this section, the DG with PFC mode is used for demonstration of the algorithm. Since voltage regulation by DG is only for the secondary network, the matrix that consists of the bottom 1040 rows and right 1040 columns of is used for the ε decomposition. In this demonstration, $\varepsilon = 0.012$. The result of this decomposition is shown in Fig. 3. It can be seen from Fig. 3 that the network is decomposed into 82 groups with a maximum of 239 nodes and a minimum of one node per group. There are a maximum of 81 DGs and a minimum of one DG per group. The number of nodes covered by the 82 groups is 647, thus 393 nodes in secondary network are not in the influence range of any DG; see Fig. 1. The information on the grouping and the sensitivity matrix can be stored in the 311 control agents when multiagent system (MAS) is used to implement the proposed method. Note that communication links between each agent in every group are needed.

C. Finding the "Nearest" DGs

It is possible that the voltage exceeds the operating limits in the largest group (group number 5 in Fig. 3), which contains nearly 250 nodes and 81 DGs. Although the size of this group is much smaller than the original secondary network that contains 1040 nodes and 311 DGs, the LP problem for voltage regulation could still have 81 variables and around 500 constraints if all the DGs of this group are involved in the voltage regulation process. However, for this kind of (large) groups, it is highly probable that not all DGs have a strong coupling with all nodes in the group. Therefore, it is necessary to find the "nearest" DGs when voltage violations happen to decrease the size of the LP problem. In the proposed method, the "nearest" DGs to a node are those DGs which range of influence covers the node. The number of nodes that are under the influence of a given DG is shown in Fig. 4; it varies from 2 to 28. When a voltage violation happens in a node or nodes, only the corresponding "nearest" DG or DGs will perform the voltage regulation function. This decreases the size of the LP problem from 81 variables and around 500 constraints to only a few variables with a few dozen constrains. Obviously, for a group containing only a few DGs, the nearest DGs would be all the DGs in the group.

D. Simulation of Optimal Distributed Voltage Control

Each DG control module has stored the following information: a) the sensitivity coefficient that describes how its DG affects the network transformers primary and secondary voltage magnitudes and voltage phase angles (obtained from the sensitivity matrix Λ); b) the sensitivity coefficient that describes how its DG affects the voltage of nodes within its influence range (from sensitivity matrix Λ after the application of the ε decomposition); c) DGs in the same group; d) acceptable voltage limits (0.95–1.05 p.u. in this paper). Also it is assumed that there exists communication links between all DGs in the same group.

For a more effective voltage control strategy two constraints can be added to (10) and (11) to prevent the network protectors from tripping. The LP program I and II can be described as:

LP I: Objective function for the DG in the PFC mode

$$\text{Max: } Min\{x_i\} \text{ (for overvoltage)} \quad (12)$$

$$\text{Min: } Max\{x_i\} \text{ (for undervoltage)} \quad (13)$$

subject to:

$$\begin{cases} V_l \le V_0 + \Lambda_{VQ} \cdot x \le V_u \\ 0 \le \theta_{p0} + \Lambda_{\theta_p Q} \cdot x - (\theta_{s0} + \theta_{shift} + \Lambda_{\theta_s Q} \cdot x) \\ x \le Q_{surplus} \end{cases} ; \quad (14)$$

LP II: Objective function for the DG in the UPF mode

$$\text{Max: } Min\{x_i\} \tag{15}$$

subject to:

$$\begin{cases} V_l \le V_0 + \Lambda_{VP} \cdot x \le V_u \\ 0 \le \theta_{p0} + \Lambda_{\theta_p P} \cdot x - (\theta_{s0} + \theta_{shift} + \Lambda_{\theta_s P} \cdot x) \\ x \le P_{surplus} \end{cases} . \tag{16}$$

The above LP I and II do not correspond to the standard LP problem. However, they can be solved by a modification that brings them to the standard LP problem by adding a slack variable y. LP I is used as example for modification shown in (17) and (18).

$$\text{Max: } y (\text{for overvoltage}) \tag{17}$$

subject to:

$$\begin{cases} V_l \le V_0 + \Lambda_{VQ} \cdot x \le V_u \\ 0 \le \theta_{p0} + \Lambda_{\theta_p Q} \cdot x - (\theta_{s0} + \theta_{shift} + \Lambda_{\theta_s Q} \cdot x) \\ x \le Q_{surplus} \\ x_i \ge y, (i = 1 \sim N, N \text{ DGs are involved}) \end{cases} . \tag{18}$$

The method for converting LP II to a standard LP problem is the same as that for LP I. A step increase of load is used to simulate DG disconnection from the network. Assume that two DGs at load 79 and 171 disconnect from the network, which is simulated by increasing load at bus 1670 from 0.0036 p.u. to 1.003 p.u. and bus 1627 from 0.0025 p.u. to 0.5025 p.u. This causes the voltage at 10 nodes on the secondary network to exceed the operating limit as shown in Fig. 5. In the small insert of Fig. 5, the horizontal line is the lower limit of normal operation (0.95 p.u.). It is clear from Fig. 5 that two DG disconnections cause problems in the neighborhood, but have very little influence on other areas. Based on the ε decomposition results, 6 nodes with voltage beyond their limit are in group number 5 and another 4 nodes are in group number 10. Therefore, the voltage control problem is divided into 2 LP problems. The information on these two LP problems is shown in Table I. Optimal results can be found for the above two LP problems, as shown in Fig. 6. The voltage profile of the secondary network before and after the control action is shown in Fig. 7. It is clear from Fig. 7 that by increasing reactive power generation, the nodes with voltage lower than 0.95 p.u. can be successfully controlled to bring them back to an acceptable voltage (higher than 0.95 p.u.). The methodology is identical for DGs working in the UPC mode. For the same voltage drop case studied above the distributed optimal control results are as follows: With $\varepsilon = 0.009$, the secondary network of the system is decomposed into 55 groups. The number of nodes and DGs contained in each group are shown in Fig. 8. In total, all groups contain 593 of the 1040 secondary network nodes. Based on the above ε decomposition results, all 10 nodes are in group number 10. Therefore, the voltage control problem is modeled in only one LP problem. The information on this LP problem is shown in Table II. Optimal DG active power generation adjustments and voltage profile are shown in Figs. 9 and 10.

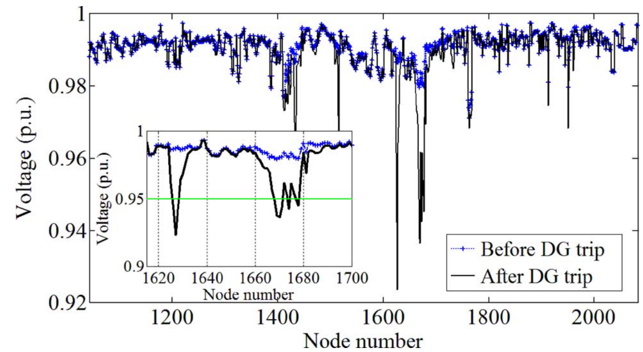

Fig. 5. Secondary network voltage profile before and after DG tripping.

TABLE I
INFO. ABOUT 2 LP PROBLEMS REPRESENTING THE VOLTAGE CONTROL

Item	Group 5	Group 43
No. of nodes with voltage beyond normal limits	6	4
No. of DGs involved in the control	21	10
No. of nodes involved in the control	54	20
No. of transformers involved in the control	2	0

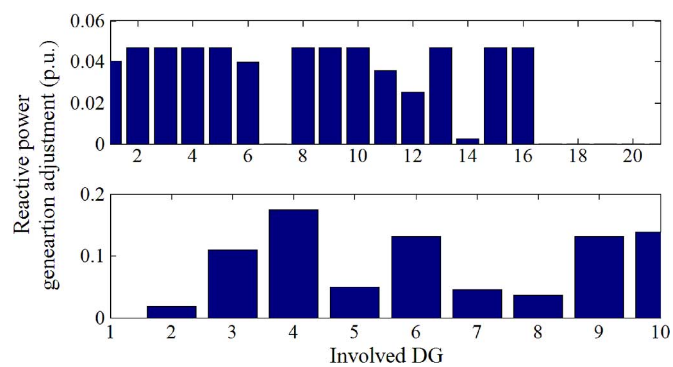

Fig. 6. Reactive power generation adjustments of involved DGs.

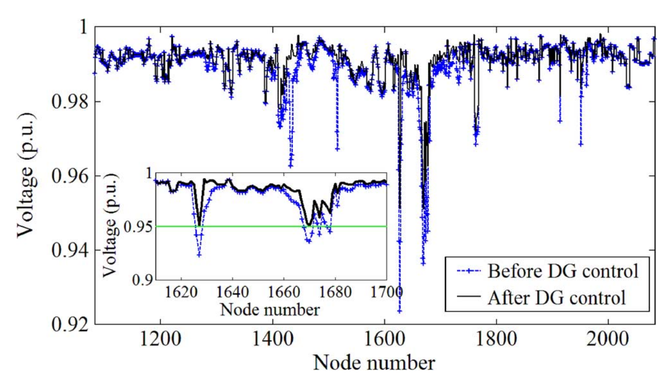

Fig. 7. Secondary network voltage profile before and after control.

IV. ANALYSIS OF THE CONTROL ALGORITHM

It is clear that the ε decomposition determines the extent of the distribution of the control algorithm. Decomposing the entire network into small groups also means reducing the network communication needs to small groups. This also reduces the wide area measurements of voltage to local measurements. In this section, different ε values are applied with the algorithm. Decomposition results and control simulation results, including successful control rate, LP size, and loss information before and after control, and minimum surplus capacity requirement for voltage regulation are shown and discussed.

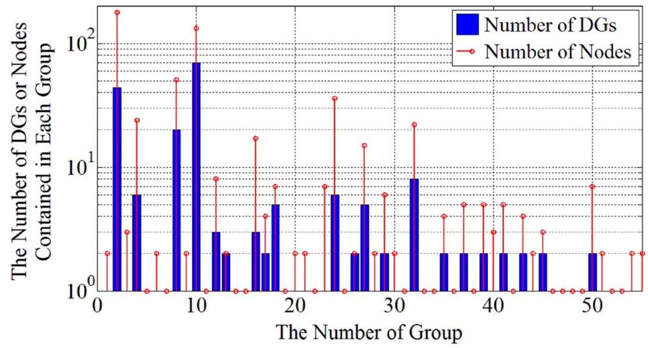

Fig. 8. Results of the ε decomposition for the matrix $\Lambda_{VQ}\varepsilon = 9e-3$.

TABLE II
INFO. ABOUT 1 LP PROBLEM REPRESENTING THE VOLTAGE CONTROL

Item	Group 10
No. of nodes with voltage beyond normal limits	10
No. of DGs involved in the control	27
No. of nodes involved in the control	63
No. of transformers involved in the control	2

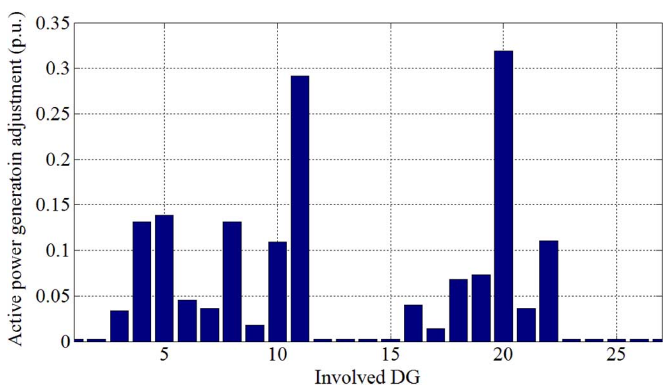

Fig. 9. Active power generation adjustments of involved DGs in UPC modes.

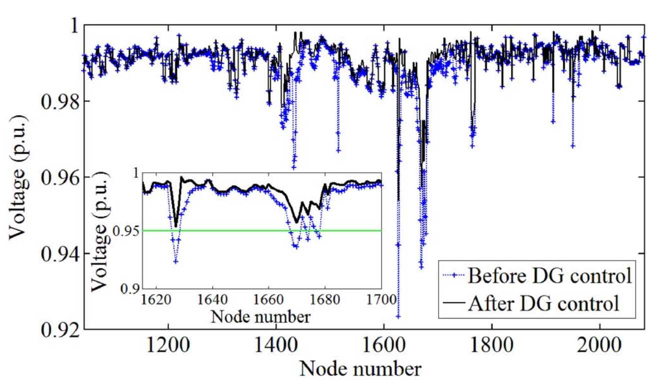

Fig. 10. Secondary network voltage profile before and after control.

A. ε Decomposition With Various ε Values

Let $\varepsilon = 0.004$, 0.006, 0.008, 0.010, 0.012, 0.014, and 0.016 for DGs in both PFC and UPC modes. The decomposition results are shown in Table III. From the results, it can be seen that with larger values of ε, the network is decomposed into more groups. When $\varepsilon = 0.004$, for the DGs in the PFC mode, the network is decomposed into 19 groups with maximum 917 nodes contained in one group and covering all nodes in the network. This is exactly the network from where we started. This also means that the voltage control is centralized, which requires global communication in every subnetwork. However, for the DGs in the UPF mode with the same ε value of 0.004, the decomposition results show that there are 20 groups with maximum 769 nodes contained in one group and covering 80% of all nodes in the network. This means that with $\varepsilon = 0.004$, some couplings between DGs and nodes have been neglected. It also corroborates the known fact that the influence of active power on voltage is smaller than that of reactive power. From two columns labeled "No. of DG groups" in Table III, we can see that with a higher ε, more groups are obtained. Also columns "% of nodes covered in DG groups" show that with a higher ε value, DG groups cover fewer nodes of the network. This is because larger ε makes decomposition to neglect more couplings. Columns "Max. No. of nodes covered in one group" show the number of maximum nodes covered in one group. With larger ε fewer nodes are covered, which means that reduced communication requirements are needed in each group. However, this does not mean that larger ε is better. With $\varepsilon = 0.016$, a maximum of 47 nodes for DG in PFC mode and a maximum of 48 nodes for DG in UPF mode are covered in one group. However, at this time, the network coverage is only 40.8% for DGs in the PFC mode and 34.5% for DGs in the UPF mode.

B. Voltage Control Results With Various ε Values

In this section, 284 voltage control simulations are performed for each ε. Each time, a different load in the largest subnetwork (containing 284 loads) is used for simulation as described in Section IV-A. For each set of 284 simulations, there are 78 cases with voltage violation which need to be regulated by DG. The purpose of the simulation is to study how different ε values affect the voltage control. In order to analyze the control algorithm itself and avoid influence from the surplus capacity of the involved DG, it is assumed that every involved DG has enough surplus capacity to control the voltage. Statistical summaries of simulations results, including total loss before and after control for all successful control cases are shown in Tables IV and V.

The statistical information in Tables IV and V shows critical results of successful control for different ε values. In the second column of Tables IV and V, the denominator stands for the number of times that the voltage dropped below 0.95 p.u. after load addition. The numerator stands for the number of times that successful voltage control was achieved. Data in the second column also indicates that without limitation on the surplus capacity of a DG, distributed control performs almost as well as centralized control ($\varepsilon = 0.004$). The third column describes the number of different groups with voltage lower than 0.95 p.u. when adding 1 p.u. active power to the load nodes. It also indicates the number of different DG groups involved in the control of the 284 simulations. From the data in the third column, one can find that with larger ε the network is decomposed into more subnetworks. The fourth and fifth columns give the average size of the LP problem for voltage control; the larger the value of ε, the smaller the size of the LP problem for voltage control. This also indicates that with larger ε, the voltage control becomes a "localer," i.e. local voltage measurements with local communication acting on local DGs.

TABLE III
RESULTS OF ε DECOMPOSITION WITH DIFFERENT VALUES OF ε

ε	DG with PFC mode			DG with UPF mode		
	No. of DG groups	% of nodes covered in DG groups	Max. No. of nodes covered in one group	No. of DG groups	% of nodes covered in DG groups	Max. No. of nodes covered in one group
0.004	19	100	917	20	80.6	769
0.006	24	97.7	888	35	70.9	456
0.008	34	85.9	596	47	61.3	382
0.01	57	74.6	412	65	54.3	142
0.012	82	62.2	239	75	46.7	108
0.014	80	52.0	163	72	37.6	58
0.016	85	40.8	47	75	34.5	48

TABLE IV
STATISTIC RESULTS OF VOLTAGE CONTROL SIMULATIONS FOR DG IN PFC MODE

ε	Successful control rate	No. of groups with under-voltage	Average No. of nodes involved	Average No. of DGs involved	Average power loss before control (p.u.)		Average power loss after control (p.u.)	
					Active power	Reactive power	Active power	Reactive power
0.004	76/78	1	131.4	21.3	0.2937	0.8551	0.3536	0.9135
0.006	75/78	1	67.9	15.3	0.2938	0.8551	0.3504	0.9078
0.008	74/78	2	42.3	12.4	0.2943	0.8555	0.3516	0.9063
0.010	71/78	5	31.2	10.4	0.2952	0.8559	0.3534	0.9042
0.012	71/78	12	23.4	8.6	0.2914	0.8440	0.3493	0.8901
0.014	70/78	15	18.7	7.1	0.2956	0.8561	0.3524	0.8984
0.016	69/78	25	14.9	6.1	0.2956	0.8562	0.3527	0.8976

The last four columns provide system average active and reactive power losses during the voltage control. Based on these results, it can be found that system power losses increase when DG works in PFC mode and power losses decrease when DG works in UPF mode. These can be seen as another criterion for designing DG based voltage regulation system. From Tables IV and V, it is obvious that there is trade-off when deciding between DG control in UPF mode or PFC mode. DG with UPF mode decreases power losses; however, DG with PFC mode have higher success control rate. Loss minimization is beyond the scope of this paper since the objective function becomes nonlinear. Future research will be carried out in this direction.

Note, however, that ε cannot be arbitrary large. The reasons are: 1) with large ε values too many nodes will be neglected in the decomposition progress; 2) there will be fewer DGs involved in the control, which means that for the same voltage control case, it is possible to require enlarged surplus capacity of each DG involved. In order to show how involved DG adjustments change with different ε values, the relative minimum surplus capacity is obtained. A case with $\varepsilon = 0.006$ is taken as a base case and different scenarios with feasible solutions are used for comparison. Fig. 11 gives the different ε values and the relative least capacity needed for each case. From Fig. 11, it can be seen that with larger ε more surplus capacity of involved DGs is needed. Hence, with the DG surplus capacity data, Fig. 11 can be also used to decide how distributed control could be utilized in the network.

C. Implementation of the Control Algorithm

The proposed algorithm can be implemented via multiagent system (MAS). First, control agents and measurement agents are installed in the network, and are divided into several groups based on system ε decomposition results, which also determine the communication links between agents. Then, interaction rules are set for each control agent and measurement agent in every subnetwork. For very large distribution systems with millions of nodes, it is possible that after the application of the ε decomposition, the subnetworks still have several thousand nodes. Therefore, the size of LP could be very large. Under these conditions, the fast and memory-efficient projection methods described in [18] and [19] are ideal solvers for the LP model. Currently, the main obstacle for implementation of proposed algorithm is that a communication system between the nodes is not existent today. Note, however, that with the method of our paper the extent and complexity of the communication system is substantially reduced in comparison with a fully connected system as suggested by previous authors.

V. CONCLUSIONS

Voltage regulation in distribution networks using DG is increasingly gaining importance. This paper has introduced a DG-based optimal distributed algorithm for voltage regulation applicable to large heavily-meshed distribution networks. The algorithm is based on the ε decomposition of the network sensitivity matrix. This neglects weak couplings between DGs and nodes while keeping the strong couplings. Therefore, for control purposes, a large network is functionally subdivided into a number of smaller subnetworks. Also whole network global communication is replaced by subnetwork local communication.

All control of DGs is installed with information about their neighboring DGs, node voltage profile in the influence range, sensitivity coefficients, and reference voltage. Based on this information the voltage regulation can then be performed effectively on the small subnetworks and the voltage of every node can be kept within its normal operating conditions by the neighboring DGs only. This is achieved by optimizing the DGs' active or reactive power. When DGs operate in the unity power

TABLE V
STATISTIC RESULTS OF VOLTAGE CONTROL SIMULATIONS FOR DG IN UPF MODE

ε	Successful control rate	No. of groups with under-voltage	Average No. of nodes involved	Average No. of DGs involved	Average power loss before control (p.u.)		Average power loss after control (p.u.)	
					Active power	Reactive power	Active power	Reactive power
0.004	68/78	1	70.5	15.9	0.2946	0.8558	0.2613	0.7935
0.006	67/78	2	42.6	12.0	0.2943	0.8558	0.2582	0.7938
0.008	66/78	5	28.9	9.1	0.2946	0.8559	0.2599	0.7962
0.010	65/78	12	20.9	7.4	0.2952	0.8562	0.2612	0.7976
0.012	63/78	20	15.0	6.3	0.2959	0.8564	0.2581	0.7964
0.014	63/78	23	13.3	5.9	0.2959	0.8564	0.2583	0.7970
0.016	61/78	26	12.8	5.7	0.2972	0.8572	0.2592	0.7973

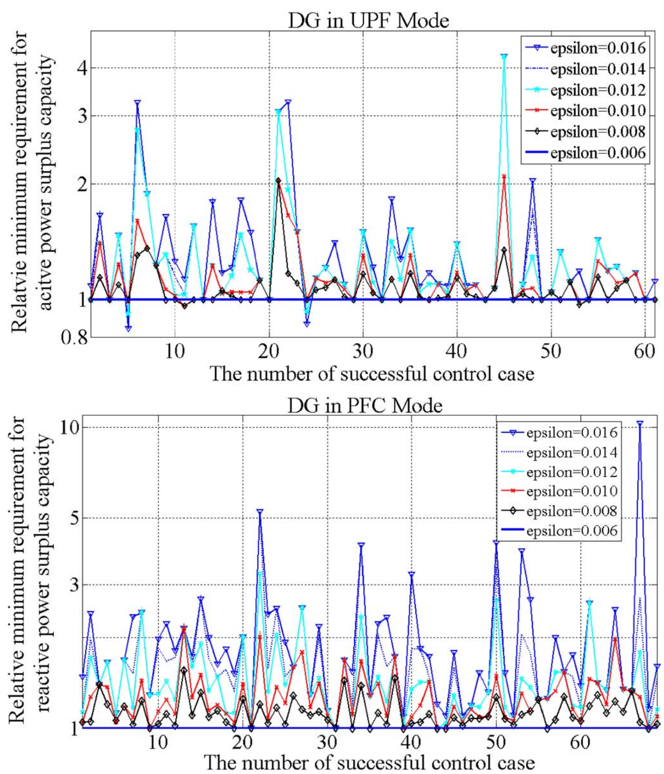

Fig. 11. Relative least surplus capacities for various values of ε (epsilon). DGs in the UPF mode (top) and DGs in the PFC mode (bottom).

$$\begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{pmatrix} 1 & 0.4 & 0.8 & 0 \\ 0.3 & 1 & 0 & 0.3 \\ 0 & 0.2 & 1 & 0 \\ 0 & 0.8 & 0 & 1 \end{pmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} \end{array} \quad (19)$$

$$\begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{pmatrix} 1 & 0 & 0.8 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0.8 & 0 & 1 \end{pmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} \end{array} \quad (20)$$

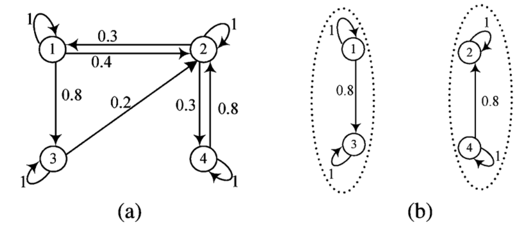

Fig. 12. (a) Network diagram before ε decomposition. (b) Network diagram after ε decomposition.

factor control mode, the proposed method optimizes the active power output of each DG. When the DGs operate in the power factor control mode the reactive power of each DG is optimized. Under these conditions, only local measurements and communications are needed.

The voltage regulation problem is solved using a linear programming method. The proposed algorithm is capable of minimizing the size of the linear programming problem for a large test network from more than 100 constraints to around a dozen. The proposed algorithm also decreases the number of variables of the linear programming problem. Additionally, it is able to find the optimal solution that makes every involved DG to "try its best" to regulate voltage. The objective function can be set to other linear functions, such as giving a different cost to every DG. Therefore, the method can also be used to minimize the cost of regulating voltage, for example. The paper has also proposed a way to determine a proper ε value.

APPENDIX

A simple network with a 4×4 sensitivity matrix is used to illustrate the ε decomposition algorithm. The matrix, which can be assumed to be Λ_{VP}, is shown in (19). The diagram of the network is shown in Fig. 12(a). $\varepsilon = 0.5$ is used for decomposition. Thus, the elements equal or smaller than 0.5 are set to zero. The new matrix is shown in (20) which is in (4). From (20), one can clearly appreciate the remaining couplings between the nodes; however, the subnetworks cannot be seen easily. In this paper, DFS [17] is used to analyze the network topology of matrix (20). The results are shown in Fig. 12(b); as we see nodes 1 and 3 are coupled and so are nodes 2 and 4, but there are no couplings between the two subnetworks.

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